



Simulation-driven experimental hypotheses and design: a study of price impact and bubbles

Simulation: Transactions of the Society for Modeling and Simulation International
2023, Vol. 99(6) 599–620
© The Author(s) 2022
DOI: 10.1177/00375497221138923
journals.sagepub.com/home/sim



Francesco Cordon¹, Caterina Giannetti¹ , Fabrizio Lillo^{2,3}
and Giulio Bottazzi⁴

Abstract

A crucial aspect of every experiment is the formulation of hypotheses prior to data collection. In this paper, we use a simulation-based approach to generate synthetic data and formulate the hypotheses for our market experiment and calibrate its laboratory design. In this experiment, we extend well-established laboratory market models to the two-asset case, accounting at the same time for heterogeneous artificial traders with multi-asset strategies. Our main objective is to identify the role played in the price-bubble formation by both *self-impact* (i.e., how trading orders affect the price dynamics) and *cross-impact* (i.e., the price changes in one asset caused by the trading activity on other assets). To this end, we vary across treatments the possibility of traders of diverting their capital from one asset to the other, thereby artificially changing the amount of liquidity in the market. To simulate different scenarios for the synthetic data generation, we vary along with the liquidity the type of trading strategies of our artificial traders. Our results suggest that an increase in liquidity increases the cross-impact, especially when agents are market-neutral. Self-impact, however, remains significant and constant for all model specifications.

Keywords

Market-impact, cross-impact, synthetic experimental asset markets, multi-assets, bubbles, trader heterogeneity

1. Introduction

Since the seminal work of Smith et al.,¹ (henceforth, SSW) experimental asset markets proved to be a very powerful tool to analyze bubble-crash patterns, which turns out to be a very persistent phenomenon in the laboratory under different settings (e.g., the research studies^{2–6}). A delicate aspect of every experimental analysis is the formulation of hypotheses. Before conducting any laboratory experiment, researchers have to state the assumptions that they are going to test in the laboratory (a process often known as preregistration process often known as pre-registration.⁷) and calibrate their market design, e.g., decide the number of participants and sessions trading time. All these steps, which have to be planned carefully, produce non-trivial issues for researchers. A normally followed route is to conduct preliminary pilot experiments, which are quite costly but allow the researcher to collect preliminary data. Pilot sessions can also be used to formulate hypotheses if not supported by theoretical modeling. However, the situation might be extremely challenging with novel experiments, as previous setup and knowledge can be of limited help. In

that event, an alternative path which follows in this paper is to rely on a simulation-based approach. More precisely, starting from a specific experimental market design, we derive synthetic data which allows us to track the price dynamics across different treatments, formulate our experimental hypotheses and calibrate our design.

Our experimental design builds on the well-established setup of SSW (e.g., the literature⁸): we extend it to the two-asset case as our main objective is to identify the role that price impact, in its two components of *self-* and *cross-*impact, has on the price-bubble mechanism. Price impact describes the relation between orders and price changes, which plays a crucial role in real financial markets

¹Department of Economics and Management, University of Pisa, Italy

²Department of Mathematics, University of Bologna, Italy

³Scuola Normale Superiore, Italy

⁴Sant'Anna School of Advanced Studies, Italy

Corresponding author:

Caterina Giannetti, Department of Economics and Management,
University of Pisa, Via Cosimo Ridolfi 10, Pisa 56121, Italy.
Email: caterina.giannetti@unipi.it

dynamics, leading to flash crashes or instability events occurring in very short time scales (e.g., intraday) in which liquidity plays a fundamental role, e.g., the Flash Crash of 6 May 2010 and the Treasury bond flash crash of 15 October 2014.^{9–12} *Self-impact* describes the price changes triggered by orders on the same stock,^{13–15} while *cross-impact* captures the effects that price changes trigger on other assets (see, for example, the literature^{16–19}). Indeed, as observed during the Flash Crash of 2010, a cascade of instabilities might affect a large set of assets and the entire market very rapidly,⁹ e.g., as a consequence of the execution of assets portfolio orders and more generally on the commonality of liquidity across assets.²⁰

To identify these effects, we vary across treatments the possibility of traders to move capital between markets, allowing in one case traders to divert money from one asset to the other (treatment *T2-Unique*), while in the other (*T1-Separated*), they have a separated portfolio for each asset. Compared to empirical work (e.g., the literature²¹), our experimental analysis has the main advantage of having a complete control over market dynamics, which allows us to explore how price impact evolves during distinct trading phases. In particular, we can keep track of the fundamental values of each asset, an unobservable variable in real data. In addition, the simplicity of our design (i.e., having only two stocks) allows us to identify and estimate cross-market relationships with relative ease compared to empirical analysis, often hampered by the estimation of very large matrix.

However, due to the novelty of our analysis, it is hard to precisely formulate our main hypotheses and calibrate the experimental design. Wan and Hunter²² showed that simulated markets might generate similar patterns observed in experimental asset markets. Furthermore, several works attempt to explore the impact of artificial traders on financial markets by employing different agent-based simulations analyses (e.g., the literature^{23–25}). Similarly, we follow this route by setting up a series of agent-based models replicating our experimental design, which allows us to generate synthetic data to analyze the price dynamics under different scenarios. More precisely, we extend the model of Duffy and Ünver⁸ in the two-asset case incorporating heterogeneous agent strategies.

The Duffy and Ünver's model⁸ is one of the first (along with the literature^{26,27}) to compare the results from financial market with artificial traders with those of experimental markets with human traders. In particular, they propose an agent-based computational approach with near-zero artificial traders to replicate the experiments of SSW. They also analyze the impact of intelligent traders with differing fundamental motivations on agent-based simulations of financial markets bring more insight into the micro-structural dynamics that work against market efficiency.

To generate the synthetic data according to the Duffy and Ünver model, we first need to adapt it to our case, thereby extending it to the two-asset case. In addition, to proxy the behavior of the human participants in the laboratory and improve our ability to study real hybrid financial markets, in the spirit of Baghestanian et al.,⁵ we introduce specific heterogeneous artificial traders which follow different investment strategies (i.e., being market-neutral or directional traders) (Modern markets are hybrid markets. It has been estimated that algorithmic traders are involved in up to 70% of the total trading volume.⁶). Importantly, our exercise not only serves to confirm the price-bubble formation and cross-impact effects, but also to calibrate our experimental market and formulate relevant hypotheses about the drivers of market impact. In particular, we employ the order flow imbalance measures of Cont et al.²⁸ to retrieve estimates of self- and cross-impact matrix during the boom phase and study how cross-impact changes with respect to other trading periods and between treatments (In contrast to da Gama Batista et al.,¹¹ we do not incorporate the market impact directly in the price dynamics, and we estimate it without imposing any prior impact model.). The final objective will be to replicate the same type of analysis once our experimental data are collected in the laboratory to assess the role of human behavior in the market and its consequences on price impact.

The paper is structured as follows. In section 2, we present our experimental design. In section 3, we illustrate the structural price model providing extensions to the multi-asset case. In section 4, we present the results of structural price models investigating the price dynamics with different model specifications, and in section 5, we report the market impact estimates based on the previous model specifications. In section 6, we derive the hypotheses we want to investigate and validate with laboratory data. In section 7, we conclude this paper.

2. Market environment design

Our design consists of a market where there are J agents which interact in $T = 15$ trading period and trade $M = 2$ assets. Each trading period is composed of 180 s, where a trader can submit an ask or bid price for the two assets.

Following the research studies,^{1,4,5} we define the fundamental value of an asset as the discounted dividend cash flows plus a terminal value. For asset i , we denote with \bar{d}_i the expected dividend payment for asset i , i.e., the average dividend paid by asset i , and with TV_i its buy-out (or terminal) value, i.e., the terminal payoff paid by i at the end of the last trading period T . Then, at time t , the fundamental value for asset i is provided by:

$$FV_{t,i} = (T - t + 1) \cdot \bar{d}_i + TV_i$$

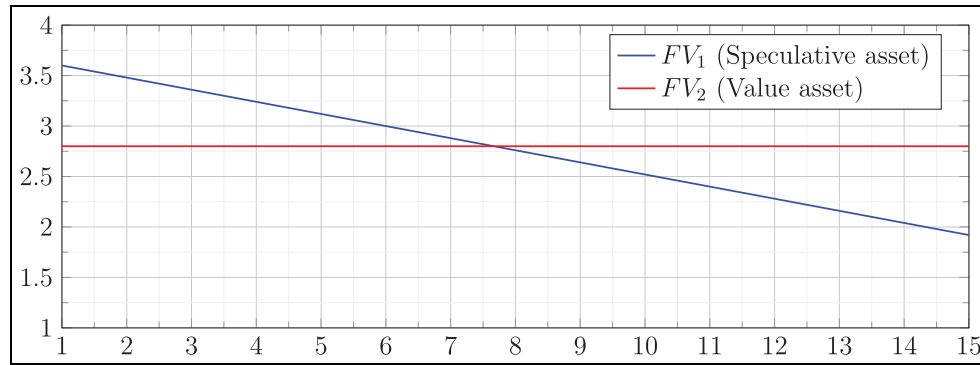


Figure 1. Average fundamental values for the two assets. The blue (red) lines represent the value of the *Speculative* (*value*) asset.

For both assets, the dividend process is described by a Bernoulli process as follows (Negative dividends represent holding costs, see Kirchler et al.⁴). The dividends distribution of the first asset P_1 is drawn by a uniformly distributed random variables with support $d_1 = \{0, 0.1, 0.16, 0.22\}$ and terminal value equal to 1.80. Therefore, $FV_{t,1} = (T - t + 1) \cdot 0.12 + 1.80$. The dividends of the second asset are defined on the support $d_2 = \{-0.2, -0.1, 0, 0.1, 0.2\}$ where $d_{2,TV} = 2.80$. We remark that if $\bar{d}_t > 0$ the fair value $FV_{t,i}$ is decreasing as $t \rightarrow T$. Thus, the first asset has a declining fundamental value with an average decreasing trend by 0.12 for each period (see Figure 1). However, the second asset has an expected dividend value of zero with a terminal value of 2.8. For convention, we name asset 1 the *speculative* asset and asset 2 as the *value* asset (Even if we adopt the same terminology of Caginalp et al.,² we did not use their “speculative relation” to establish if an asset is more speculative than another (see Caginalp et al.² for further details).).

Table 1 reports a summary of statistics for the two fundamental values. The fundamental values intersect each other around round 8. Figure 1 displays the average fundamental values for the two assets.

2.1. Market treatments

In our experiment, traders will have the opportunity to buy and sell assets in each period via a continuous double-auctions open limit order book (see, for example, the literature^{1,4,29}). At the beginning of the experiment, each participant is endowed with two fictitious asset units, i.e., asset 1 and asset 2, and a cash balance of US\$5.85 in total (In the laboratory, we will use our experimental currency, the ECU.). Our experimental market will be a hybrid one with artificial agents and human participants. Each order is for only one share (as in Fink et al.²⁹). The order book is empty at the beginning and at the end of a period, and it is anonymous, i.e., the identity of the trader submitting an order is concealed. In particular, in all treatments, there

Table 1. Summary statistics of the fundamental values for the two assets.

Dividends	$d_1 \in \{\$0, \$0.10, \$0.16, \$0.22\}$	$\bar{d}_1 = \$0.12$
Initial value	$FV_{1,1} = 3.6$	
Terminal value	$FV_{t,1} = (T - t + 1) \cdot 0.12 + 1.80$ $TV_1 = 1.80$	
Dividends	$d_2 \in \{-\$0.20, \$-0.10, \$0, \$0.10, \$0.20\}$	$\bar{d}_2 = \$0$
Initial value	$FV_{1,2} = 2.80$	
Terminal value	$FV_{t,2} = 2.80$ $TV_1 = 2.80$	

will be two types of traders, human participants and noise traders, along with market-makers. Noise traders are “near-zero-intelligence” agents, and can be likened to inexperienced subjects in the market (see Duffy and Ünver⁸). A market-maker is an agent who, on a continuous and regular basis, proposes prices at which he is ready to buy and sell a given asset.³⁰ That is, as in real financial markets, the role of market-makers in our experiment is to provide liquidity (see in detail section 3.4). (Market-makers are said to be part of 70% of the electronic trades in the United States (40% in the European Union (EU) and 35% in Japan) (see Guéant et al.³¹). Some of them are “official,” i.e., there is an agreement with an exchange for maintaining fair and orderly markets (e.g., the Designated Market Makers on the New York Stock Exchange (NYSE)) while others are just acting as liquidity providers without any obligation to do it (e.g., high-frequency traders) (see Guéant.³⁰) Having more liquidity in the market, i.e., more transactions from both types of artificial agents, is essential for our purpose as it allows us to precisely estimate the effect of market impact by increasing the number of available observations while keeping track of agents’ actions. On top of that, having market-makers in addition to noise agents results in a price volatility more in line with what is observed in real financial markets (as described in details in section 4). To sum up, in our experimental market, we will have:

1. human traders that cannot go short and do not know which players are posting the order. They will play a relevant role in the laboratory experiment;
2. noise traders buy and sell according to the traditional Duffy and Ünver model, and post quotes following a homogeneous Poisson process. They can be considered as “near-zero intelligent” agents. They have the same initial inventory as human participants (see section 3);
3. market-makers post bid/ask quotes at the beginning of the sessions following the Avellaneda/Stoikov model (see Ho and Stoll³² and Avellaneda and Stoikov³³ and more information in section 3). In contrast to other participants, they can go short. Their main role is to provide liquidity to the market, thereby increasing market efficiency.

To isolate the effect of price impact, we conduct two main treatments: the first one (*T1-separated*) features a market in which traders have two separated portfolios (i.e., participants cannot divert the money of one asset to the other one); the second one (*T2-Unique*) features a market in which traders have a unique portfolio (i.e., the money can be freely invested in each of the two assets with no restrictions). Thus, treatment *T1-separated* is similar to Caginalp et al.² except for traders’ inability to freely move capital across markets (i.e., stocks). Indeed, in *T2-Unique* (with independent orders) traders can freely divert their capital from one asset to the other. Thus, comparing these two treatments allow us to identify the effect of changes in liquidity on cross- and self-impact. Importantly, for both treatments, asset dividends are placed in the respective portfolio. Kirchler et al.⁴ show how merging the savings account for dividend cash with portfolio cash implies an increasing Cash/Asset (*C/A*) ratio, which in turn generates an increase in the available liquidity for traders.

Since in our simulation-based analysis, we do not consider the presence of human agents, in the spirit of Baghestanian et al.,⁵ we setup different investment strategies for artificial traders which can be used as proxies for the human participants’ behavior. In particular, we will consider either *directional* or *market-neutral* players, i.e., players that either place orders in the same side in both markets or in the opposite side in the two markets (see section 3.3). Indeed, this study will serve as a future base to evaluate the behavior of financial markets featuring human participants as active players.

Thus, in section 4, we additionally examine the *T2-Unique* treatments in which players follow one of the two factor-investing style strategies, namely, *T2-Unique-Directional* and *T2-Unique-Market-Neutral*, or a combination of the two, namely, *T2-Unique-Heterogeneous*. Finally, to assess the role of market-makers, we integrate them in previous setup, namely, *T2-Unique-Heterogeneous-MM*.

3. Structural agent-based model for order book dynamics

In this section, we first describe the Duffy and Ünver model.⁸ We then extend this model to our multi-asset environment, additionally considering the existence of market-makers. Finally, we specify a heterogeneous agent-based model when traders follow different factor-investing style strategies.

3.1. The Duffy and Ünver model

In the Duffy and Ünver market model, J agents trade the same asset P_1 in T trading periods. The asset pays a random dividend at the end of each period t . Each trading period t is composed of S submission rounds, where traders can place their orders, following a double-auction market mechanism with continuous open-order book dynamics. Specifically, in each submission round, agents have to determine their position, i.e., whether they buy or sell the asset, and the amount they are willing to pay or receive for the asset (i.e., a quote). A Bernoulli variable decides the traders’ position in each submission round. The probability to be a buyer decreases in each trading period so that in the last trading sessions, traders are more prone to sell. This condition allows the Duffy and Ünver model to capture the same liquidity dynamics observed in Smith et al.¹ and is named the *weak-foresight assumption*. The quote of a trader’s order is then determined by the weighted sum of the previous period’s prices and a random value proportional to the fundamental value. Duffy J and Ünver⁸ introduce this randomness in traders’ quote to capture traders’ uncertainty about the fundamental value, while the weighting parameter, the so-called anchoring parameter, indicates that agents are more likely to post quotes close to the previous period’s prices (see also Baghestanian et al.⁵) and represents a crucial parameter to explain the price-bubble shape.

Therefore, during the round s in the trading period t , an agent can place buy (bid) or sell (ask) orders. Agents can submit a bid or ask quotes for a unit of the asset, see also Smith et al.¹ where standard bid and ask improvement rules are employed.

We now report the main model specification we have implemented from the Duffy and Ünver model:

- We allow a trader to place an order in all the bid and ask sides, i.e., if he is a buyer (seller), the agent can place a bid (ask) price which is not necessarily greater (smaller) than the current best bid (ask) price. However, the quotes cannot be unbounded since they must satisfy the traders’ inventory condition (see point below). Furthermore, a trader can have only one outstanding limit order, and an agent

cannot be in the bid and ask side in the book simultaneously.

- For each trading time period t , at the beginning of each round s , the trading priority is assigned by a permutation of participants. Then, once a trader is selected, a Bernoulli variable, $\mathcal{B}(\pi_t)$, determines the trader's position (buyer or seller) for the submission round. So, if in a trading period t and at round s , a trader is selected to be a buyer (seller) and has an open ask (bid) position in round $s - 1$, the trader will not submit any order in the round s . We refer to this condition as the *one-side condition*.
- The probability to be a buyer, during the trading period t at submission round s , of an agent j is given by π_t . We assume the *weak-foresight* assumptions of Duffy and Ünver,⁸ i.e., the probability to be a buyer is decreasing among the trading periods:

$$\pi_t = \max\{0.5 - \varphi t, 0\}, \text{ where } \varphi \in \left[0, \frac{0.5}{T}\right)$$

As stressed in Duffy and Ünver,⁸ we choose $\varphi > 0$ to get consistent results with the experimental data. A positive φ implies a gradual increase of excess supply toward the end of the market and so it contributes to the reduction in mean transaction prices. In particular, its primary role is to reduce the transaction volume over time consistent with the experimental data (see Smith et al.¹ and Duffy and Ünver.⁸) This assumption is crucial for the Duffy and Ünver model to generate the observed crash patterns in the laboratory market (By considering the heterogeneous model of Baghestanian et al.,⁵ the weak-foresight assumption can be dropped as we did in a companion paper. The price dynamics of the Baghestanian et al. model heavily depends on a more complex parameterization of agents' strategies compared to simplest Duffy and Ünver model. Moreover, since their model is calibrated with experimental data where only one asset is considered, instead of presenting preliminary results using their parameters, we prefer to postpone the analysis with the Baghestanian et al. model when our experimental data are available.)

- At the end of each trading period t we compute the mean traded price, $\bar{p}_{t,i}$, which is publicly available. The mean traded price \bar{p}_t is defined as the average of the mean of round prices \bar{p}_t^s . If the volume of transactions, i.e., number of shares traded in period t round s is denoted by vol_t^s , then:

$$\bar{p}_t^s = \begin{cases} \frac{1}{vol_t^s} \sum_{h=1}^{vol_t^s} p_{t,h}^s & \text{if } vol_t^s > 0 \\ p_t^{b-a,s} & \text{if } vol_t^s = 0 \end{cases}$$

where $p_t^{b-a,s}$ is the mean bid–ask spread price, and $p_{t,h}^s$ is the price of the h th unit traded in period t of session s . Then, $\bar{p}_t = 1/S \sum_{s=1}^S \bar{p}_t^s$ represents a measure of the market price of a share, see Duffy and Ünver⁸ for further details.

- At the beginning of the market session, each trader j has an endowment of cash x^j and a quantity of the asset y^j . Then, a buyer (seller) j in period t at round s can place (Where we also consider the one-side condition.) a bid (ask) quote if enough cash balances $x_{t,s}^j > 0$ (share quantity $y_{t,s}^j > 0$) is available in his account. Thus, trader j places a quote following a convex combination of the previous period mean traded price \bar{p}_{t-1} and a random quantity $u_{t,s}$. This random price is proportional to the current expected fundamental value drawn from a uniform distribution with support $[0, \kappa \cdot FV_t]$, where $\kappa > 0$. This noise captures the possibility that agents can make some decision errors. So, if j is a buyer, j can place a bid price given by:

$$b_{t,s}^j = \min\{(1 - \alpha)u_{t,s} + \alpha\bar{p}_{t-1}, x_{t,s}^j\}$$

and if j is a seller, he can place a ask price given by:

$$a_{t,s}^j = (1 - \alpha)u_{t,s} + \alpha\bar{p}_{t-1}$$

where $\alpha \in (0, 1)$ is the so-called *anchoring* parameter. The anchoring parameter plays a crucial role in the price-bubble formation, since prices will necessarily increase at the beginning to decrease as the fundamental value decreases, with the number of sellers increasing over time. As stressed in Duffy and Ünver,⁸ this kind of explanation for the price-bubble mechanism holds regardless of φ . As we will see, when $\varphi = 0$ the price will continue to get a “hump-shaped” path with no decrease in transaction volume:

- When the submitted bid (ask) price is greater (smaller) than or equal to the current best ask (bid) price, the unit is sold at the current best ask (bid) price. At the end of each trading period t , after the round S , the order book is completely cleared (We simply clean out the two sides of the order book without executing limit orders.), where dividends are paid out and we update the agent cash accounts. In particular, at the beginning of the first round, $s = 1$ in the trading period $t + 1$ the book is initially empty (We will introduce market-maker agents to ensure enough liquidity to traders.). We employ a real-time adjustment rule, i.e., during a trading period t in a round s , any executed orders are immediately executed and the cash and share accounts are, respectively, adjusted.

Even if the particular architecture specification would appear to influence the simulation results, as remarked in Duffy and Ünver,⁸ the results are insensitive to the type of order book convention and structure.

3.2. Two-asset extension

We now introduce the generalizations of the previous model to our experimental design. To extend the Duffy and Ünver model in a the two-asset market, we may simply specify two model specifications to the two assets P_1 and P_2 . Therefore, we have two order books with the relative parameters, $\kappa_i, \alpha_i, \varphi_i$. However, we have to carefully set the time priority. Specifically, we consider two different time priority for the two books related to the two assets. Therefore, in a given round s , the two time priorities of book 1 and book 2 are embedded and executed alternatively (We have tested in preliminary analyses other execution ordering and we did not find any particular change in simulations results.). More precisely, if $\tau(\cdot)$ denotes the time priority (i.e., a permutation of the N traders) for the round s , we first execute the first order for book 1, $\tau_1(1)$, and then that of book 2, $\tau_2(1)$. Then, we consider the second ones, $\tau_1(2)$ and $\tau_2(2)$, so that in events time, we have $\tau_1(1) > \tau_2(1) > \tau_1(2) > \tau_2(2) > \dots > \tau_1(i) > \tau_2(i) > \tau_1(i+1) > \tau_2(i+1)$, and so on.

The main feature of the multi-asset scenario is the design of a specific multi-asset trading strategy which traders can implement. We are going to explain the details of this feature in the next section.

3.3. Heterogeneous agent-based model: factor-investing styles

The near-zero-intelligence agents of Duffy and Ünver can be essentially viewed as a prototype of noise traders in real financial market. Therefore, to understand how different trading strategies can induce significant cross-impact effects in a multi-asset scenario, and to better proxy the behavior of human traders, we introduce artificial agents with different strategies mimicking factor-investing style (see, for example, Li et al.³⁴).

To do that, we assume that the traders read a signal to buy or sell assets following one of the assets i . We assume that agents follow the speculative asset, and we denote the signal as $s_i \in \{-1, 1\}$, where 1 means that s_i is a buy signal and -1 a sell signal. The probability of reading a buy or sell signal is modeled by π_i , i.e., the probability to be a buyer or a seller for asset i .

Then, in order to introduce heterogeneity in our population, we design a percentage of agents following one of the two market factors v_j . A *directional* trader places orders on both assets following the directional market factor, i.e.,

$v_D = [1, 1]^T$. However, a *market-neutral* agent will place orders following $v_M = [1, -1]^T$. Therefore, when an agent reads the market signal s_i , he decides the position on the asset i and if the trader is a directional will place the same order side on the other assets, i.e., the position on both assets are described by the product $s_i \cdot v_D$, while in the market-neutral case, the agent will place an opposite order side on the other asset, e.g., his order will be determined by $s_i \cdot v_M$.

Therefore, we consider three classes of agents: directional, market-neutral, and noisy traders. Each trader chooses a position following the speculative asset P_1 and decides the position on the value asset P_2 using a market factor for the directional and market-neutral agents, while the noise trader randomly selects the two-asset positions. As stated above, the positions on asset one are chosen using the probability the weak-foresight assumptions of Duffy and Ünver.⁸ The quotes, $b_{i,s}$ and $a_{i,s}$ are placed randomly for all the traders following the specification of Duffy and Ünver.⁸

Remark 3.1. An agent who follows the directional or market-neutral vectors could be interpreted as a stylization of a general factor-investing strategy. Indeed, the two market factors v_D and v_M are the eigenvectors of a general symmetric matrix:

$$\Lambda = \begin{bmatrix} 1 & \lambda \\ \lambda & 1 \end{bmatrix}$$

and this matrix could represent the correlation of the two assets or, more interestingly, the cross-impact matrix.

3.4. Agent-based model with market-makers

To ensure sufficient liquidity at all trading periods and to make our laboratory design closer to typical real market architecture, we include market-maker agents, which place bid/ask quotes in an opening session before round $s = 1$ for all trading period t and act as liquidity providers for all other rounds. This should provide price and order book dynamics in line with what observed in real market sessions.

We employ the Avellaneda–Stoikov (AS) market-making model (see Ho and Stoll³² and Avellaneda and Stoikov³³), where market-makers place optimal quotes in order to maximize the expected (Costant Absolute Risk Adversion (CARA)) utility criterion within a finite time horizon T in an order book. We consider the setting of Guéant et al.³¹ where market-makers have a maximum authorized inventory Q , which can be, in contrast to traders, either long or short. Furthermore, Guéant et al.³¹ proved that the optimal bid and ask quotes of the market-maker in the Avellaneda–Stoikov model are given, respectively, by:

$$S^{b*}(t, p, q) = p - \frac{1}{\kappa} \ln\left(\frac{v_q(t)}{v_{q+1}(t)}\right) - \frac{1}{\gamma} \ln\left(1 + \frac{\gamma}{\kappa}\right)$$

$$S^{a*}(t, p, q) = p + \frac{1}{\kappa} \ln\left(\frac{v_q(t)}{v_{q-1}(t)}\right) + \frac{1}{\gamma} \ln\left(1 + \frac{\gamma}{\kappa}\right)$$

where p is the current value of the reference price (the mid-price), γ is the market-maker’s risk-aversion, κ characterizes the price sensitivity of market participants and the functions $v_q(t)$, $|q| \leq Q$, which make the market-maker’s optimal quotes depend on its inventory, denoted by q , and they are defined as:

$$(v_{-Q}(t), \dots, v_Q(t))' = \exp(-M(T-t)) \times (1, \dots, 1)'$$

$$M = \begin{bmatrix} \alpha Q^2 & -\eta & 0 & \dots & \dots & \dots & 0 \\ -\eta & \alpha(Q-1)^2 & -\eta & 0 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 & -\eta & \alpha(Q-1)^2 & -\eta \\ 0 & \dots & \dots & \dots & 0 & -\eta & \alpha Q^2 \end{bmatrix}$$

where $\alpha = \kappa/2\gamma\sigma^2$ and $\eta = A(1 + \gamma/\kappa)^{-(1 + \kappa/\gamma)}$, σ is the volatility of the asset, and A characterizes its liquidity.

In the original work of Avellaneda and Stoikov,³³ the market-maker’s inventory is modeled by the differences of two point processes which model the trading activity of traders, i.e., the number of assets that have been, respectively, bought and sold. In our setting, a market-maker updates his inventory when a trader hits one of his quotes at time t . Right after, the market-maker updates his optimal quote according to the new inventory (The other parameters, A , γ , κ), the remaining trading period time $T - t$ and reference price p_t , where the volatility parameter σ_t is fixed as the intra-trading period volatility of the previous trading period, i.e., the volatility observed in $[t - 1, t]$.

In order to have a symmetric initialization of the order book, we select J_{MM} heterogeneous market-makers distinguishing them in terms of risk-aversion.

We remark that even if the Avellaneda and Stoikov model is time continuous, the model can be reinterpreted in a discrete way as also observed by Guéant et al.³¹ Therefore, we may use its discretization to produce reliable market-makers’ quotes.

3.5. Two-asset agent-based market model

We now summarize our agent-based model for the two-asset market experiments. Recall that our market is designed as a continuous double-auction market as in Duffy and Ünver⁸ and agents are distinguished between traders and market-makers. Market-makers post bid and ask quotes at the beginning of the session following the

Avellaneda/Stoikov model, see section 3.4, and they will eventually update their quotes if traders accept one of their quotes. Therefore, market-makers agents will provide and guarantee the necessary liquidity to the market in each trading round so that a trade can occur. In contrast to traders, they can go short.

However, traders are the only participants who can actively participate in the market, i.e., they can accept quotes from other participants, which then generate a trade. They can only have one outstanding position for each asset and each quote is for one share, where standard bid/ask improvement rules are applied. Depending on the market treatments, traders can follow one of the factors investing trading strategies of section 3.3.

Regarding the sequentiality, at the beginning of each trading period, t , an opening session is executed where market-makers place their quotes, characterizing the initial liquidity offer for the two assets order books. Then, round sessions start and the time priorities for the two books are generated, τ_1 and τ_2 . Then, traders can place orders, and they are sequentially called following the two time priorities, where we embed τ_1 and τ_2 as explained in section 3.2. An order of the trader can be posted in the book or executed. In the last case, a trade occurs where the counterpart can be another trader or a market-maker. When the counterpart is another trader, then both agents update their inventories and their quotes are removed from the book. However, when the counterpart is a market-maker, the trader and market-maker’s inventories are updated and the market-maker updates quotes according to the new inventory following the Avellaneda and Stoikov model. At the end of submission round S , the book is cleared and dividends are paid, while at the end of the trading period T , the terminal value is paid for each remaining share in traders’ inventories for both assets.

Table 2 and Algorithm 1 report a summary of the simulation design.

3.6. Simulation setup

We design a market experiment where each market session consists of $T = 15$ trading periods and where each period lasts 180 s. Based on previous experimental evidence (see, for example, Palan³⁵), we hypothesize that a trader could submit an order for the two assets every 30 s. Therefore, we set up in simulation $S = 6$ rounds for each trading period.

We remark that in $T1$ treatment, agents have separate portfolios, where traders have initial endowments of US\$2.925 and two units of asset for the two portfolios. In $T2$ treatment, each trader has a merged portfolio with an initial endowment of US\$5.85 and two units for each asset (In the first simulations, we divide traders in three classes depending on different initial endowments among $T1$ and $T2$ treatments, as in Smith et al.¹ and Duffy and Ünver.⁸

Table 2. Simulation design summary.

Agents		Market
Traders	Market-Makers	
<ul style="list-style-type: none"> Initial inventory is composed of cash and assets. Can only have one outstanding position for each asset and each order is for one share. Standard bid/ask improvement rules. Buy and sell according to Duffy and Ünver⁸ with the extension provided in section 3.2. Can follow one of the factor-investing trading styles, see section 3.3. 	<ul style="list-style-type: none"> Post bid and ask quotes at the beginning of the sessions following the AS model. Can go short. 	<ul style="list-style-type: none"> A market session of $T = 15$ trading periods, each one divided in $S = 6$ submission rounds. In each round s, a trader can post bid/ask quotes. The counterpart can be any another trader or market-maker. Trade orders are executed only by traders (i.e., in trader/trader order, the inventory is updated for both, in market-maker/trader, the inventory is updated for the trader while the market-marker updates according to Avellaneda/Stoikov model) Dividends are paid at the end of each period t, and the inventories are updated.

Algorithm 1. Pseudo code of market design. The algorithm illustrates the sequence of the different operations between traders and market-makers and the different market phases, i.e., from the opening session when dividends are paid. We distinguish traders from market-makers since traders are the only ones who can actively participate in the market, i.e., they can accept quotes from other traders. Market-makers in our design can only supply liquidity to the market.

Participants of the market: traders, market-makers.

```

for ( $t = 1 : T$ ) {
  Trading period  $t$  starts;
  run opening session;
  Market-makers place quotes in the order book, characterizing the initial liquidity offer;
  for ( $s = 1 : S$ ) {
    Round trading session  $s$  starts;
    Traders place orders following our extension of the DU model;
    The time priorities for the two books are selected,  $\tau_1$  and  $\tau_2$ ;
    Traders are sequentially called following the two time priorities, where we embed  $\tau_1$  and  $\tau_2$  as explained in section 3.2;
    An order of the trader can be posted in the book or executed. In the last case, the order generates a “trade” where the counterpart can be another trader or a market-maker;
    if A “trade” is of the type Trader/Trader then
      Both agents update their inventories;
    end
    if A “trade” is of the type Trader/Market-maker then
      The trader’s inventory is updated and the market-maker updates his quotes according to the new inventory following the Avellaneda and Stoikov model;
    end
    Round trading session  $s$  ends;
  }
  Trading period  $t$  is ends;
  Dividends are paid for both assets.
}
The terminal value is paid for each remaining share in traders inventories by each asset.

```


Table 3. Model specification: $\pi_{t,i} = \max\{0.5 - \varphi_i, 0\}$ is the probability to be a buyer or a seller for the asset i .

	φ_i	α_i	κ_i
$i = 1$	0.01643	0.8499	4.1946
$i = 2$	0	1 - 0.8499	2

Bid and ask quotes are given by, $b_{t,s} = \min\{(1 - \alpha_i)u_{t,s}^i + \alpha_i\bar{p}_{t-1,j}, x_{t,s}\}$ and $a_{t,s} = (1 - \alpha_j)u_{t,s}^j + \alpha_j\bar{p}_{t-1,i}$, where $u_{t,s}^i$ is drawn by a uniform distribution with support $[0, \kappa_i d_{t,i}]$.

However, we did not observe any significant differences in terms of price dynamics and market impact estimates. Thus, in T1, traders cannot divert cash of one asset in the other. We consider one type of treatment T1 (*T1-Separated*) and three types of T2 treatments, where agents place orders independently for the two assets and are either all directional (*T2-Unique-Directional*), all market-neutral (*T2-Unique-Market-Neutral*), or a combination of the two (*T2-Unique-Heterogenous*). Finally, we consider two heterogeneous model specifications in which we additionally consider the presence of $J_{MM} = 10$ market-makers. For each simulation treatment, we run $N = 100$ simulations with a total of $J = 33$ traders (The number of participants is chosen similar as much as possible to those of our future experiment in the laboratory.).

The order type (buy or sell) is decided by $\pi_{t,1}$, the probability related to the asset 1. This means that agents, in some sense, follow the price of the asset 1 and decide to buy or sell the asset 2 depending only on $\pi_{t,1}$ (As complementary treatments, we consider the case when the order side is decided fixed by $\pi_{t,2}$, i.e., traders follow the value asset. In other words, the directional and market-neutral cases correspond trivially to the case when the agents place an order following a unique Bernoulli variable. The results are available upon request, since we observe no relevant findings.). To determine the quotes, we set for the speculative asset P_1 , the agents' parameters based on the estimates of Duffy and Ünver,⁸ i.e., $\kappa_1 = 4.1946$, $\alpha_1 = 0.8499$, and $\varphi_1 = 0.01643$. These parameters were estimated by Duffy and Ünver⁸ in order to replicate SSW experiments, so we might expect that the price dynamics of P_1 will exhibit the typical bubble shape of market experiments (Duffy and Ünver⁸ have calibrated their model to reproduce the SSW experiments.). Since, $\kappa_1 = 4.1946$ traders will post on average twice the fundamental value of P_1 even if they will put more weight on previous price. This will generate the same hump-shaped pattern of Smith et al.,¹ where traders start to trade at a low price level and subsequently generate an upward trend which is finally eliminated by large-scale selling orders posted by traders due to the weak-foresight assumption ($\varphi_1 > 0$). However, since the dynamics of the value asset should be aligned with its fundamental value (see Kirchler et al.⁴), the parameters of the asset 2 are set in a complementary way with respect to that of asset 1.

Thus, for asset 2, we set $\kappa_2 = 2$, $\alpha_2 = (1 - 0.8499)$, $\varphi_2 = 0$. A value $\kappa_2 = 2$ would force agents to trade at the intrinsic value on average. The weight α_2 given to the “anchor” \bar{p}_{t-1} is complementary to that for asset 1. This implies that agents place an order on the asset 2 with bid/ask prices which are close to the dividend fair value $d_{t,2}$ than past prices. Finally, since we set $\varphi_2 = 0$, we expect to observe no imbalance between demand and supply as the one observed for asset 1. We summarize the model parameters in Table 3.

4. Simulation results on price dynamics

In this section, we analyze price dynamics generated by the various treatments according to the basic models. We also marginally consider the case when market-makers are included, even if their role is primarily to provide enough liquidity to human agents when we will go to the laboratory. However, artificial (noise) traders in the structural model are designed independently from the presence of market-makers. We thus expect they will have a significant role in the laboratory sessions when humans and artificial traders play together. As a result, in this first analysis, we mainly present price-dynamics results concerning models where market-makers are not considered, although in the next section 5, we will also present and discuss the estimates of cross-impact when market-makers are included as well.

We first analyze the effect of changing traders' liquidity on price dynamics by comparing *T1-separated* and *T2-Unique*. In line with the experimental results of Kirchler et al.,⁴ we observe an amplification in price levels due to the increase in C/A , when both dividends are collected in the same portfolio for each trader, i.e., in *T2-Unique* treatment (see Figures 2 and 3).

Then, to understand how different trading strategies can impact prices, we examine two T2-treatments where traders follow one between *directional* and *market-neutral* market factors. We observe that directional traders generate an undervaluation effect on the value asset P_2 , contrary to market-neutral agents, which produce an overvaluation effect on P_2 (see Figures 4 and 5).

Finally, we investigate the effect of market-makers by comparing T2-treatments where all the previous features are implemented, i.e., considering heterogeneous agents, of the previous treatments and where market-makers are included in the market. The liquidity provided by market-makers does not seem significant alter the price dynamics of both assets, although we observe that at the beginning of the trading session, the dynamics of P_1 is more aligned with its fundamental value, Figure 7, compared when market-makers are absent, Figure 6.

For the sake of clarity, we summarize each treatment results in the following:

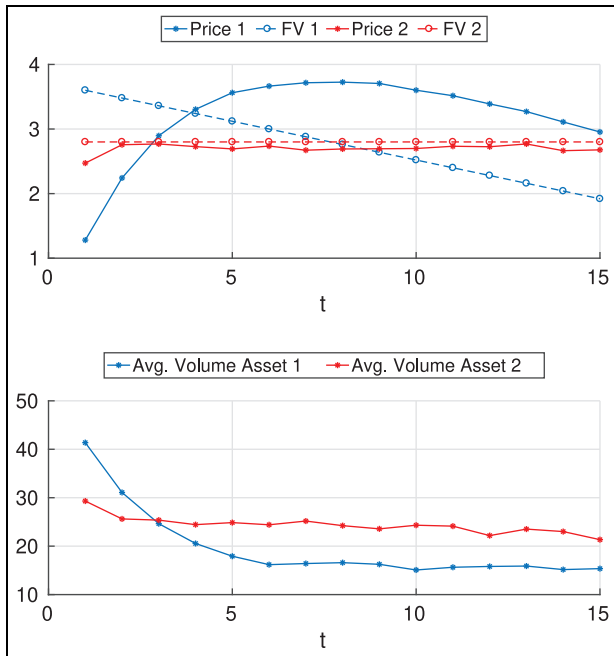


Figure 2. Mean transaction price and average volume of shares traded among the trading periods for *T1-Separated*.

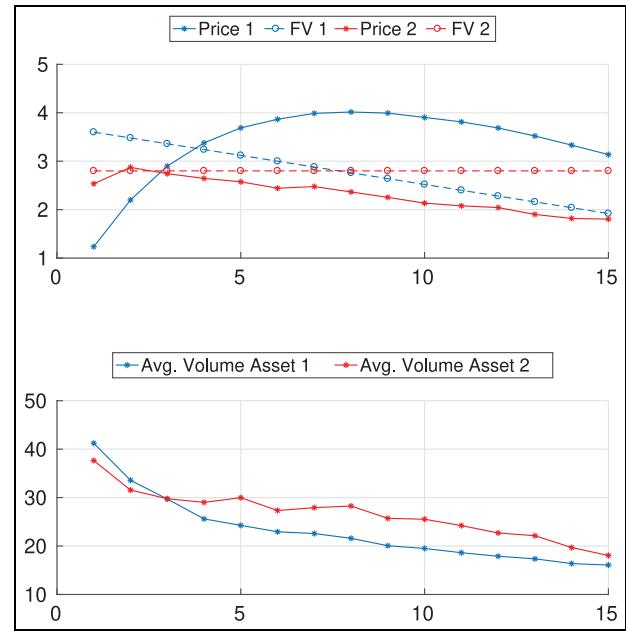


Figure 4. Mean transaction price and average volume of shares traded among the trading periods for *T2-Unique-Directional* (following P_1).

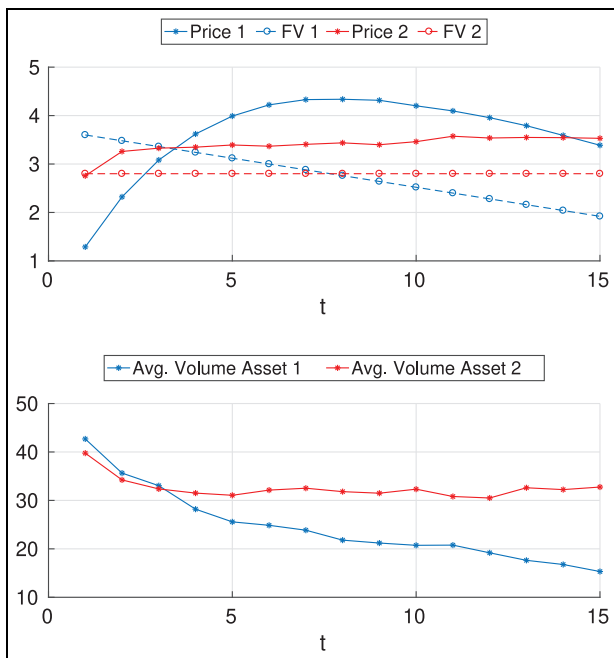


Figure 3. Mean transaction price and average volume of shares traded among the trading periods for *T2-Unique-Independent*.

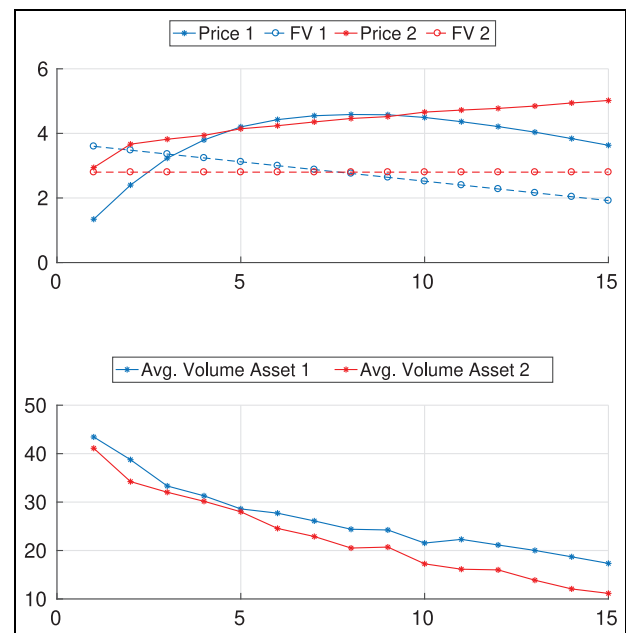


Figure 5. Mean transaction price and average volume of shares traded among the trading periods for *T2-Unique* treatment with market-neutral orders (following P_1).

1. *T1-Separated*: this is our base treatment in which agents have separate portfolios. We basically observe the same behavior as in Duffy and Ünver,⁸ which based on SSW experiments, i.e., a price bubble emerges for the speculative asset 1

(see P_1 in Figure 2). Moreover, we observe the same price shape of Kirchler et al.⁴ for the second asset since the fundamental value is constant (see P_2 in Figure 2).

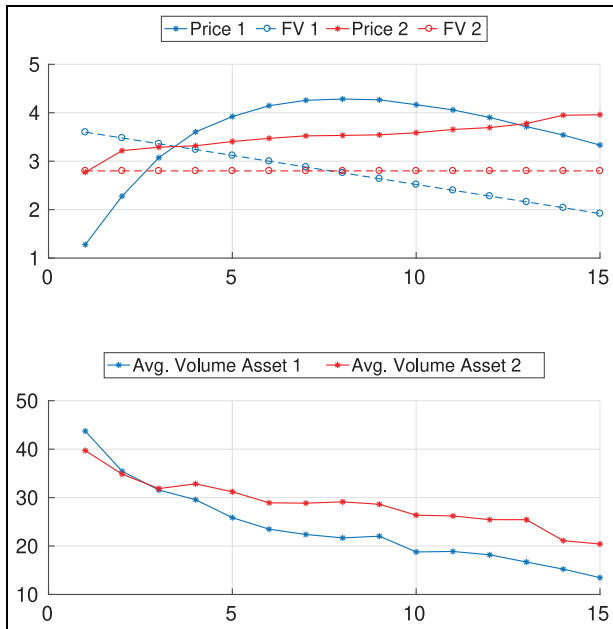


Figure 6. Mean transaction price and average volume of shares traded among the trading periods for *T2-Unique-Heterogeneous*. There are 33% noise traders, 33% directional, and 33% market-neutral traders.

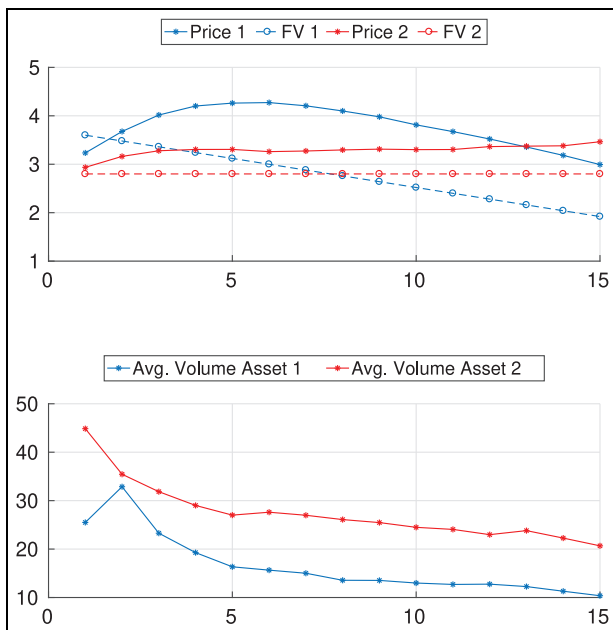


Figure 7. Mean transaction price and average volume of shares traded among the trading periods for *T2-Unique-Heterogeneous* with 10 market-makers. There are 33% noise traders, 33% directional, and 33% market-neutral traders.

2. *T2-Unique* with *independent orders (T2-Unique-Independent)*: in this treatment, agents' cash accounts are merged into a single one for the two assets considering the same endowments as in

T1-separated. Agents place order in each of the two assets independently. As expected, in line with the results observed by Kirchler et al.,⁴ we observe an amplification on price levels, e.g., see the price for the value asset P_2 , due to the increase in C/A since both dividends are collected now in the same portfolio for each trader, which generates an overall increase in cash (see Figure 3).

3. *T2-Unique* with *directional orders (T2-Unique-Directional)*: this treatment is similar to the previous one although agents' orders now follow the speculative asset 1 and place order in both assets in the same position, i.e., using the directional market factor. This treatment is also similar to the experiment of Kirchler et al.,⁴ when part of the investment capital is diverted from the value asset toward the speculative one. Indeed, in addition to observing a price bubble emerging for the speculative asset 1, we observe a price reduction for the value asset (see Figure 4).
4. *T2-Unique* with *market-neutral orders (T2-Unique-Market-Neutral)*: this is a complementary treatment to the previous one, whereby agents place orders, following the speculative asset P_1 , for both assets in opposite positions, i.e., using the market-neutral factor. In this case, we observe a price bubble for both assets (see Figure 5).
5. *T2-Unique* with heterogeneous *directional* and *market-neutral* orders (*T2-Unique-Heterogeneous*): this treatment combines the feature of previous treatments, in which there are 33% noise traders, 33% directional traders (always following the speculative asset P_1) and 33% market-neutral traders (see Figure 6).
6. *T2-Unique* with heterogeneous *directional* and *market-neutral* orders when 10 market-makers are present (*T2-Unique-Heterogeneous-MM*): we set the market-makers parameter equal to $\kappa = 1$, $A = 50$, $Q = 20$. Each market-maker has different risk-aversion parameter γ , which is selected using an equidistant grid in $[0.5, 1]$. Results in this case resemble the one presented in point 5. Having market-makers providing liquidity does not significantly alter the price dynamics of both assets, although they are more realistic at the beginning and at the end of the experiment (see Figure 7).

As a robustness check, we also repeated treatments 3–4 where the agents follow asset 2. In other words, $\varphi_1 = 0$, so we expect that there is no decrease in volume and a subsequent price decrease in the last trading periods for P_1 . We do not observe any particular difference in price path between these treatments and the T2 with independent order, suggesting that a correlation in orders does not affect prices correlation when the agent follows the signal

from the value assets (More details are available upon request.)

To summarize, the results from our simulations suggest that a price-bubble tends to emerge in all cases for the speculative asset 1, while different dynamics appear for the price of the value asset 2. In general, we interpret the results of *T2-Unique-Independent* as those obtained by purely noise traders with no particular trading strategy. However, when we distinguish between two types of traders, namely, directional and market-neutral (i.e., *T2-Unique-Directional* and *T2-Unique-Market-Neutral*), a positive (negative) order correlation seems to imply a negative (positive) price correlation. Indeed, by construction, the order types of a directional are positively correlated (more precisely, they are equal); however, we observe a negative price correlation. This was observed by Caginalp et al.² where they concluded that there might exist a negative liquidity mechanism that induces this price correlation. This is precisely what comes out from the results of *T2-Unique-Directional*, where, however, the order correlation is positive. We remark that the quotes are independent of the two assets. Interestingly, the heterogeneous model, where the population is equally divided into three classes, noise, directional, and market-neutral (*T2-Unique-Heterogeneous*), provides equivalent results to that of the T2-independent model, where all agents are noise traders.

Finally, when market-makers are considered (*T2-Unique-Heterogeneous-MM*), the price dynamics of the speculative asset starts close to the fundamental value, hence more realistic than the previous ones. Indeed, as in SSW experiments, traders in T2-treatments, since they are inexperienced, start trading at a low value compared to the fundamental value. Then, agents gain confidence and create an upward trend which generates the typical price-bubble shape. However, this inexperience is filled by market-makers, which allows agents to trade at a more efficient price. Thus, when market-makers are included, traders may be considered more experienced than those of the Duffy and Ünver model. Interestingly, even if traders are experienced, we still observe price-bubble dynamics, which is then intrinsically characterized by the experimental market design. Furthermore, in the Duffy and Ünver model, we observe a liquidity drop when $t \rightarrow T$, the volume of transactions significantly reduces at the end of the market session. This effect is principally due to the weak foresight of traders on the speculative asset. Therefore, the liquidity generated by market-makers seems to anticipate this liquidity drop process since agents start to trade at prices close to the price-bubble peak and, therefore, with a subsequent anticipated price deflation. The parameter setting of the market-makers forces the average of their bid–ask spread to be 1.5 on average. Therefore, at the beginning of the market session, traders will mainly buy from market-makers, while at the end, traders will post (“aggressive”) quotes inside the market-makers spread,

making the liquidity provided by market-makers useless. This suggests how cash flows from traders to market-makers initially, and while at the end of the period due to the market-makers inventory risk-aversion, the liquidity provided by market-makers does not increase the transaction volumes. This explains the reduction of the number of transactions even if we have more liquidity in the market. The same price and volume dynamics are also observed for the other treatments considered.

However, if we increase the parameter κ , the market-makers’ spread will decrease and the number of transactions remains constant over the trading period (see Guéant et al.³¹). In this case, the typical price-bubble shape will no longer be observable since the price will remain constant for all trading periods at the market-makers’ mid-price.

To conclude, by analyzing the market impact among the different treatments, we notice that the *T2-Unique-Market-Neutral* exhibits the larger and significant cross-impact effect, as it is confirmed by the subsequent results.

5. Statistical price model: market and cross-impact estimation

Since the seminal paper of Kyle,³⁶ linear models for market impact are widely used to study impact of (aggregate) net order flow and price movements. We consider the popular order flow imbalance, *OFI*, measure of Cont et al.,²⁸ to estimate market impact. Roughly speaking, *OFI* represents the imbalance between supply and demand at the best bid and ask prices during a fixed time interval. In particular, after computing $OFI_t \in \mathbb{R}^M$ between period $t - 1$ to t (see Cont et al.²⁸), we estimate the following model:

$$r_t = \Lambda OFI_t + \varepsilon_t \quad (1)$$

where $r_t \in \mathbb{R}^M$ represents the assets returns, $\varepsilon_t \in \mathbb{R}^M$ is the residual term, and $\Lambda \in \mathbb{R}^{M \times M}$ is the market impact matrix. As usual the noise term ε_t is uncorrelated from OFI_t , i.e., $\text{cov}(\varepsilon_t, OFI_t) = 0$. The diagonal components of Λ represent the so-called *self-impact* coefficients, while the off-diagonal terms represent the *cross-impact* effect between the selected assets. Furthermore, due to the features of experimental data, we may infer a causal and statistical relation between fundamental values changes and OFI_t .

5.1. Market-impact estimation based on simulated data

We now compute the *OFI* and estimate market and cross-impact using simulated data for each market session. We follow Cont et al.²⁸ and the standardization of Mertens et al.³⁷ More precisely, we compute *OFI* for each trading round s among the periods. For our market design, the $OFI_{t,s,i}$ for the asset i is computed first as the difference

between of net (buy) order flow, $V_{t,s,i}^B$, and net (sell) order flow, $V_{t,s,i}^S$, where there are no cancelations at the best bid and ask, i.e.,

$$OFI_{t,s,i} = V_{t,s,i}^B - V_{t,s,i}^S$$

The net (buy) order flow is defined as $V_{t,s,i}^B = L_{t,s,i}^b - M_{t,s,i}^b$, where $L_{t,s,i}^b$ denotes the volume (number of shares) of limit buy orders at the best bid, and $M_{t,s,i}^b$ denotes the volume of market (sell) orders occurring at the best bid during round s . However, $V_{t,s,i}^S = L_{t,s,i}^a - M_{t,s,i}^a$, where $L_{t,s,i}^a$ denotes the volume of limit sell orders at the best ask, and $M_{t,s,i}^a$ denotes the volume of market (buy) orders occurring at the best ask during round s (see Cont et al.²⁸ for further details). Then, following Mertens et al.,³⁷ we standardize $OFI_{t,s,i}$ by rescaling with its standard deviation and market depth $\delta_{t,\cdot,i}$, i.e.,

$$of\hat{t}_{t,s,i} := \frac{OFI_{t,s,i}}{\delta_{t,\cdot,i}\sigma(OFI_{t,s,i})}$$

where $\delta_{t,\cdot,i}$ is defined as the average among the rounds of the average volume at the best bid and ask, $\delta_{t,\cdot,i} = 1/S \sum_{s=1}^S (V_{t,s,i}^b + V_{t,s,i}^a)/2$, i.e., depth is defined as the average of the size at the best quotes (see also Mertens et al.³⁷).

Therefore, we compute for each round s at trading time t , following Mertens et al.,³⁷ the normalized log-returns of mid-prices $r_{t,s,i}$ (by its standard deviation) and we estimate the market impact coefficients $\lambda_{t,1,i}$ and $\lambda_{t,2,i}$ for asset $i = 1, 2$ in a panel regression among round s and across simulations for each trading period t , i.e., for each trading period $t = 1, 2, \dots, T$,

$$r_{t,s,i,k} = \lambda_{t,i,1} \cdot of\hat{t}_{t,s,1,k} + \lambda_{t,i,2} \cdot of\hat{t}_{t,s,2,k} + \varepsilon_{t,s,i,k}, \quad (2)$$

$$k = 1, 2, \dots, N; s = 1, 2, \dots, S$$

so that we may obtain an estimate of market impact among the trading periods t , for asset $i = 1, 2$. The terms $\lambda_{\cdot,1,2}$ ($\lambda_{\cdot,2,1}$) measures how the order flow imbalance of asset 2 (1) impacts the returns of asset 1 (2), i.e., the cross-impact.

Figures 8 and 9 exhibit the estimates of $\lambda_{\cdot,1,1}$ and $\lambda_{\cdot,1,2}$ obtained by the regression (2) for each trading period t for T2-Unique-Independent/Neutral and T2-Unique-Heterogeneous/Heterogeneous-MM. Tables 4 and 5 report the time average estimates of market-impact coefficients among the trading periods. The results related to T1-Separated are similar to those of T2-Unique-Independent, but with a weaker cross-impact effect due to the capital restrictions which we have imposed in T1.

We observe that the estimations of the regression model (2) highlight how the self-impact remains significant and constant among the trading periods for all the model specifications considered. Moreover, we observe asymmetric cross-impact estimates. Specifically, for almost all model

specifications, the cross-impact effect λ_{21} , the impact of order-flow imbalance of asset 1 on returns of asset 2 is positive. However, the cross-impact term λ_{12} is not statistically different from zero so that the returns of asset 1 are essentially influenced by the self-impact λ_{11} . Furthermore, from Tables 4 and 5, we observe that the cross-impact terms are smaller on average than those of self-impact. Moreover, we note that market-neutral agents seem to play a relevant role in generating a positive and significant cross-impact effect λ_{12} with respect to directional traders. When agents follow the market-neutral factor, we find significant cross-impact terms of asset 1 to asset 2. We also observe a significant cross-impact term in the simulations when the two fair values intersect. Moreover, while the self-impact estimates are quite uniform among all specifications, even when market-makers are considered, the cross-impact terms are reduced by the market-makers' effect, even though the estimates remain statistically significant.

We summarize our preliminary results as follows:

1. The market-neutral investors play a relevant role in generating cross-impact effects.
2. The cross-impact tend to be asymmetric, where the impact of the liquidity of (the speculative) asset 1 on asset 2 returns are significant and positive. The liquidity of (the value) asset 2 is not relevant to explain asset 1 returns.
3. The self-impact remains significant and constant among the trading periods for all model specifications.

In Appendix 1, we repeat the previous analysis to investigate market impact during price-bubble crashes using other market liquidity measures. We employ the order imbalance (see, for example, da Gama Batista et al.¹¹) and the excess bids measures (see Selten and Neugebauer³⁸). However, we found inconclusive and insignificant results for both measures. The volume imbalance does not provide significant coefficients even for self-impact coefficients. The excess bids measure provides, in general, noisy and contrasting results. Therefore, we restrict the statistical analysis to the OFI measure.

5.2. A possible interpretation of cross-impact effect

Since we know the fundamental value for each asset, in regression (2) we consider as an explanatory variable the ratio between fundamental values. Precisely, we consider $r_{FV,t} = \log(FV_{1,t}/FV_{2,t})$ as a distance measure between the two-asset values, which in our design is positive for the first half trading period and negative in the last trading period. In particular, $r_{FV,t}$ becomes zero in the middle period when the two-asset values intersect each other, which is the region where agents will have more difficulty to disentangle

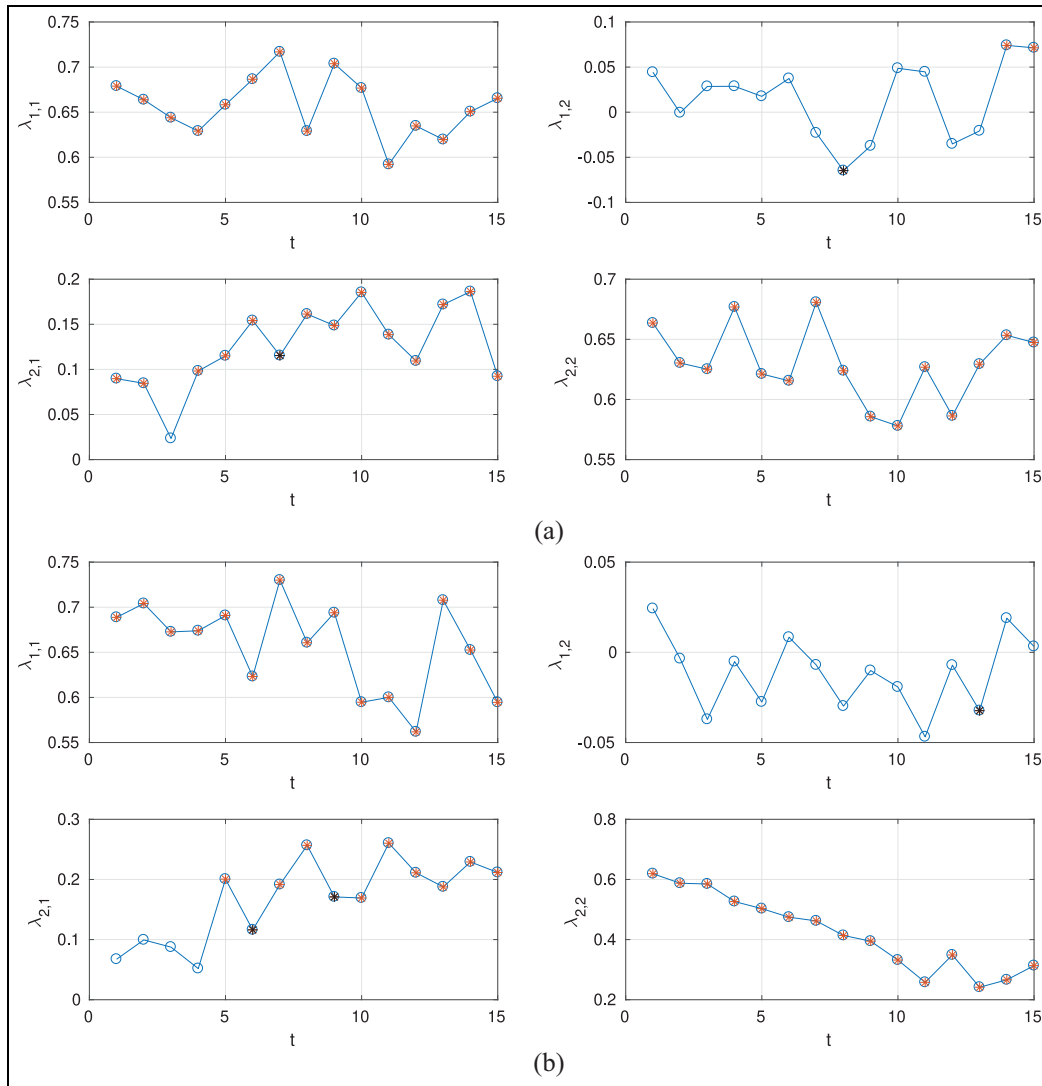


Figure 8. Market-impact estimates using regression (2). Values with orange (dark) star are significant at 5% (10%) level using HAC standard errors. (a) Estimates for treatment T2-Unique-Independent and (b) estimates for treatment T2-Unique-Market-Neutral.

the two values (We expect that the confusion generated by the intersection of fundamental values will play a relevant role with human traders rather than artificial.)

Therefore, we estimate the market impact coefficients $\lambda_{t,1,i}$ and $\lambda_{t,2,i}$ for asset $i = 1, 2$, for each round s at trading time t , in a panel regression among round s and across simulations for each trading period t , i.e., for each trading period $t = 1, 2, \dots, T$,

$$r_{t,s,i,k} = \alpha_{t,i} \cdot r_{FV,t} + \lambda_{t,i,1} \cdot of\hat{t}_{t,s,1,k} + \lambda_{t,i,2} \cdot of\hat{t}_{t,s,2,k} + \varepsilon_{t,s,i,k}, \tag{3}$$

for each $k = 1, 2, \dots, N$ and $s = 1, 2, \dots, S$. Tables 6 and 7 report the average estimates of market impact and α for different model specifications. The ratio of fundamental values filters out the cross-impact effect. This result

suggests that cross-impact effects are a masked outcome of the intrinsic relations between fundamental values and prices. The price-bubble mechanism of asset 1 pushes out the price of asset 2 as an intrinsic effect. Interestingly, the self-impact are consistent to what observed in the previous analysis, and significantly positive.

We observe that α_2 estimates turn out to be quite oscillatory among trading periods. Especially during trading period 7 and 8, when $r_{FV,t}$ is close to zero, the estimates of α_2 results to be huge and oscillatory. Since this behavior is observed among the various model specifications, we report as an example in Figure 10 the estimates of α_i for *T2-Unique*-heterogeneous specification model where there are 33% directional and market-neutral agents. The behavior of α_2 explains the huge standard deviations of the average estimate of Tables 7 and 8.

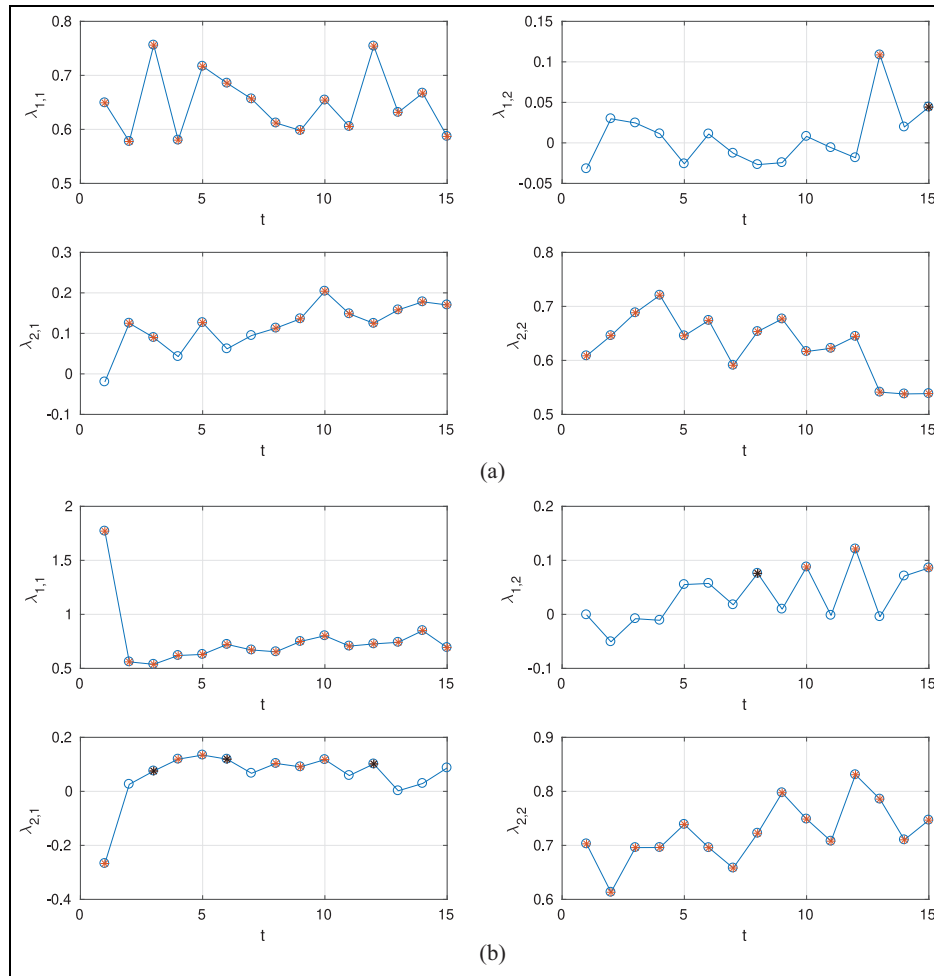


Figure 9. Market-impact estimates using regression (2). Values with orange (dark) star are significant at 5% (10%) level using HAC standard errors. (a) Estimates for treatment T2-Heterogeneous and (b) estimates for treatment T2-Unique-Heterogeneous and 10 market-makers.

Table 4. Time average, among trading period t , of market-impact estimates obtained by regression (2) for T2 treatments with homogeneous population.

	T2-independent		T2-directional		T2-Market-Neutral	
(a) Estimates for T2-Unique						
λ_{11}	0.657	(0.033)	0.639	(0.047)	0.657	(0.050)
λ_{12}	0.014	(0.042)	0.058	(0.042)	-0.011	(0.021)
Adjusted R^2	0.402		0.393		0.397	
λ_{21}	0.125	(0.045)	0.046	(0.040)	0.167	(0.067)
λ_{22}	0.630	(0.031)	0.659	(0.039)	0.422	(0.127)
Adjusted R^2	0.394		0.459		0.176	
(b) Estimates for T2-Unique with 10 market-makers						
λ_{11}	0.767	(0.295)	0.756	(0.317)	0.760	(0.285)
λ_{12}	0.034	(0.047)	0.060	(0.045)	-0.015	(0.059)
Adjusted R^2	0.419		0.380		0.457	
λ_{21}	0.046	(0.112)	0.066	(0.107)	0.088	(0.115)
λ_{22}	0.737	(0.059)	0.712	(0.060)	0.800	(0.128)
Adjusted R^2	0.317		0.317		0.312	

Standard deviations are reported in parentheses. We also report the average adjusted R^2 for each regression.

Table 5. Time average, among trading period t , of market-impact estimates obtained by regression (2) for T2 treatments with different factor-investing style agents by varying the percentage of directional (Dir.) and market-neutral (MN) traders.

Dir.	33.00%		40%		30%
MN.	33.00%		30%		40%
(a) Estimates for T2-Unique-Heterogeneous					
λ_{11}	0.649	(0.059)	0.652	(0.036)	0.653
λ_{12}	0.007	(0.037)	0.026	(0.023)	0.012
Adjusted R^2	0.381		0.399		0.385
λ_{21}	0.117	(0.057)	0.131	(0.055)	0.155
λ_{22}	0.627	(0.056)	0.634	(0.045)	0.599
Adjusted R^2	0.380		0.381		0.347
(b) Estimates for T2-Unique-Heterogeneous with 10 market-makers					
λ_{11}	0.762	(0.291)	0.765	(0.289)	0.763
λ_{12}	0.034	(0.049)	0.036	(0.043)	0.035
Adjusted R^2	0.407		0.396		0.394
λ_{21}	0.058	(0.098)	0.076	(0.062)	0.060
λ_{22}	0.723	(0.055)	0.727	(0.056)	0.742
Adjusted R^2	0.310		0.321		0.319

Standard deviations are reported in parentheses. We report also the average adjusted R^2 for each regression.

It is crucial to confirm this simulation analysis using market laboratory data. We expect to repeat the previous analysis by calibrating the models with parameters derived from experimental data and directly measuring cross-impact relations with fundamental values. Therefore, we can now formulate our main hypotheses, to be tested once experimental data has been collected.

6. Discussion: hypotheses formulation

Given the previous simulation results, we derive our main hypothesis related to the *cross-impact effect*. In particular, we expect that the uncertainty and difficulty to disentangle the two-asset values during the central phase of the experiment might trigger a significant liquidity mechanic effect by which the price of one asset is affected by orders of the other asset. We emphasize this hypothesis as follows:

H1. The uncertainty and difficulty in disentangling the two-asset values trigger a significant cross-impact effect between the two-asset prices. This effect will be stronger in treatment T2-Unique compared to T1-separated as in the former case, there are no restrictions in moving the capital from one asset to the other.

Results from our simulation analyses support this hypothesis. We observe a price-bubble emerging in the speculative asset that also affects the price dynamics of the value asset in all cases we examined (see section 4 to get an overview of the different factor-investing styles and price dynamics). Estimation of the cross-impact confirms that this effect is positive and statistically significant in most cases (see $\lambda_{2,1}$ in Figures 8 and 9 and Tables 4 and 5). If this hypothesis will be validated with laboratory data, it

will have an important policy recommendation: in order to prevent price-bubble propagations (i.e., significant price deviations from the fundamental values), the regulator could operate in the market by imposing some capital constraints that impede capitals to divert from one asset to another, i.e., as in T1 treatment.

Another hypothesis we derive concerns the cross-impact asymmetries:

H2. The liquidity mechanism which generates the price bubble does not involve a symmetric cross-impact between the two assets.

In particular, we expect a larger cross-impact of the speculative asset on the value asset, rather than the opposite. Indeed, the price bubble of the speculative asset is mainly driven by an endogenous mechanism, e.g., it does not depend on the price realization of the value asset and it is often observed in single asset experiments (see Palan³⁵ for a review). Results from our simulation results support this hypothesis. In all cases, we observe a larger and significant cross-impact of the speculative asset (see $\lambda_{2,1}$ vs $\lambda_{1,2}$ in Figures 8 and 9 and Tables 4 and 5), but never the reverse.

The third hypothesis refers to the self-price impact:

H3. The self-impact will not change significantly in treatment T2-Unique compared to T1-Separated.

In other words, we expect that the removal of liquidity constraints will only affect cross-impact without significantly affecting the self-impact. Results from our simulation results support this hypothesis (see $\lambda_{1,1}$ and $\lambda_{2,2}$ in Figures 8 and 9 and Tables 5 and 6).

Table 6. Time average, among trading period t , of market-impact estimates obtained by regression (3) for T2 treatments with homogeneous population.

	T2-independent		T2-directional		T2-Market-Neutral	
(a) Estimates for T2-Unique treatments						
α_1	0.277	(1.392)	-0.270	(0.694)	0.010	(1.055)
λ_{11}	0.642	(0.047)	0.633	(0.052)	0.633	(0.052)
λ_{12}	0.005	(0.041)	0.053	(0.044)	-0.023	(0.017)
Adjusted R^2	0.401		0.392		0.396	
α_2	-0.710	(5.979)	0.046	(0.461)	-1.304	(9.911)
λ_{21}	0.033	(0.033)	0.040	(0.048)	-0.005	(0.040)
λ_{22}	0.591	(0.030)	0.654	(0.042)	0.345	(0.141)
Adjusted R^2	0.400		0.458		0.192	
(b) Estimates for T2-Unique treatments with 10 market-makers						
α_1	-0.3117	(0.653)	-1.243	(3.151)	-0.292	(0.663)
λ_{11}	0.740	(0.292)	0.721	(0.315)	0.745	(0.278)
λ_{12}	0.013	(0.024)	0.033	(0.043)	-0.028	(0.069)
Adjusted R^2	0.419		0.380		0.457	
α_2	-0.214	(3.276)	-0.480	(2.668)	-0.512	(5.356)
λ_{21}	-0.014	(0.095)	0.032	(0.079)	-0.010	(0.096)
λ_{22}	0.695	(0.052)	0.684	(0.058)	0.704	(0.097)
Adjusted R^2	0.319		0.317		0.318	

Standard deviations are reported in parentheses. We report also the average adjusted R^2 for each regression.

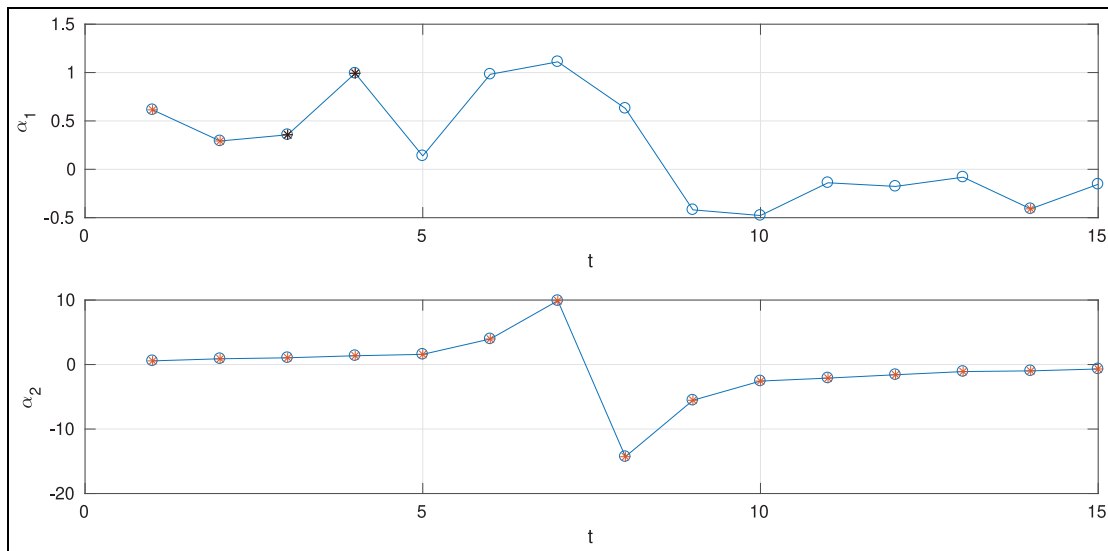


Figure 10. α estimates using regression (3) for T2-Unique-Heterogeneous model specification where there are 33% directional and 33% market-neutral agents. Values with orange (dark) star are significant at 5% (10%) level using HAC standard errors.

A further strong hypothesis which we can derive taking advantage of the experimental data is the following:

H4. Within treatments, the relationship between asset prices and fundamental values makes the cross-impact effect negligible.

This hypothesis will allow us to investigate the origin of the cross-impact effect as the result of an intrinsic relation between fundamental values. From our preliminary results, see section 5.2, the cross-impact vanishes when a distance

measure between the two-asset fundamental values is included in regression (2). If H4 results to be valid, it would point the relationships between intrinsic values as a possible explanation of the cross-impact effect, further supporting assumption H1.

7. Conclusion

Hypotheses formulation is always a crucial step when planning an experiment. This work presents a simulation-based approach to derive the hypotheses for our financial

Table 7. Time average, among trading period t , of market-impact estimates obtained by regression (3) for T2 treatments with different factor-investing style agents by varying the percentage of directional (Dir.) and market-neutral (MN) traders.

Dir.	33.00%		40%		30%
MN	33.00%		30%		40%
(a) Estimates for T2-Unique-Heterogeneous					
α_1	0.217	(0.543)	0.151	(0.912)	0.151 (0.912)
λ_{11}	0.627	(0.063)	0.628	(0.057)	0.628 (0.057)
λ_{12}	-0.004	(0.038)	0.000	(0.026)	0.000 (0.026)
Adjusted R^2	0.380		0.379		0.379
α_2	-0.635	(5.107)	-0.377	(3.812)	-0.377 (3.812)
λ_{21}	0.014	(0.046)	0.021	(0.030)	0.021 (0.030)
λ_{22}	0.584	(0.058)	0.624	(0.045)	0.624 (0.045)
Adjusted R^2	0.388		0.410		0.410
(b) Estimates for T2-Unique-Heterogeneous with 10 market-makers					
α_1	-0.641	(1.745)	-0.357	(1.959)	-1.063 (3.288)
λ_{11}	0.731	(0.286)	0.729	(0.287)	0.728 (0.283)
λ_{12}	0.005	(0.048)	0.000	(0.053)	-0.002 (0.029)
Adjusted R^2	0.407		0.397		0.394
α_2	-0.471	(4.113)	-0.552	(3.347)	-0.549 (3.786)
λ_{21}	0.003	(0.081)	0.030	(0.043)	0.010 (0.097)
λ_{22}	0.676	(0.056)	0.685	(0.050)	0.696 (0.058)
Adjusted R^2	0.312		0.321		0.320

Standard deviations are reported in parentheses. We report also the average adjusted R^2 for each regression.

market experiment, which we will validate once the laboratory data has been collected. The present analysis relies on synthetic data derived by different agent-based models which sufficiently replicate the experimental data. We first extend the agent-based model of Duffy and Ünver to our market design, in which we consider two financial assets (instead of one) and introduce factor-investing trading strategies. We also use the simulation approach to calibrate our experimental design. Furthermore, Cordoni³⁹ shows how the price-bubble dynamics of the (simulated) experimental results presented herein are robust and persistent to the parameter choice of the asset-price models.

We believe that our work contributes to the understanding of the origin and causes of cross-impact effects in the process of price-bubble formation. In particular, in a market environment where capital is relatively segmented across treatments, while price information remains free to move, cross-impact can be seen as the result of the entanglement of asset value fundamentals, triggered by the boost of the speculative asset-price bubble. Our preliminary analysis supports this hypothesis (H1): the uncertainty and difficulty in disentangling the two-asset values trigger a significant cross-impact effect between the two-asset prices. This effect is larger when there are no restrictions in moving capital across the market. Interestingly, this effect appears to be asymmetric (H2), always triggered by the boost of the speculative asset to the value

asset, while the liquidity of the second asset never influences the price bubble. This would imply a violation of dynamic arbitrage in the sense of Gatheral⁴⁰ (see, for example, the literature^{17,18,41}) which is not surprising in a market where price bubbles are present. This suggests that the price-bubble mechanism is endogenously generated by the fundamental characteristics of the speculative asset. Finally, we also observe that the self-impact is not substantially affected by the market's liquidity alteration (H3). An exception is the self-impact of asset 2 when agents are market-neutral traders. We also expect that a significant relationship between asset price and fundamental values will emerge with experimental data. In particular, our setting suggests that taking into account a distance measure between the two-asset values will explain a great part of the cross-impact effect (H4).

In conclusion, even though these preliminary results are based on simulations derived from specific agent-based models, we can draw several interesting conclusions. All the results need to be confirmed with experimental data in order to understand the relevance of human traders in generating market impact effects under the price-bubble formation process. Furthermore, relying on the experimental data, we aim to calibrate the more sophisticated Baghestanian et al.'s agent-based model, to the two-asset market to further investigate the price impact using the previous analyses setup featuring speculative or fundamental trading strategies.

Table 8. Time average, among trading period t , of market-impact estimates obtained by regression (2) using order imbalance ρ for T2 treatments.

	T2		T2-DI		T2-MI	
λ_{11}^{OI}	-0.0018	(0.231)	0.2624	(0.260)	0.4672	(0.163)
λ_{12}^{OI}	0.3735	(0.162)	-0.5308	(0.163)	0.5007	(0.143)
λ_{21}^{OI}	-0.2907	(0.295)	-0.6556	(0.348)	-0.0287	(0.278)
λ_{22}^{OI}	0.4972	(0.233)	0.3412	(0.211)	0.5904	(0.122)

Standard deviations are reported in parentheses.

T2 refers to T2 treatments with independent orders, T2-DI refers to T2 treatments with directional orders, and T2-MI refers to T2 treatments with market-neutral orders, respectively.

Appendix I

Market impact and other liquidity measures

We then repeat the previous analysis to investigate market impact during price bubble crashes using other market liquidity measure. We report the results for the homogeneous agents model specification. We employ the order imbalance (see, for example, da Gama Batista et al.¹¹) and the excess bids measures (see Selten and Neugebauer³⁸). The order volume imbalance is defined as $\rho_t = V_t^b - V_t^a / V_t^b + V_t^a \in [-1, 1]$, where V_t^b (V_t^a) are the total volume at time t of limit of buying (selling) orders (see da Gama Batista et al.¹¹). Similarly, as for the volume imbalance (see, for example, Cartea et al.⁴²), there is strong buying pressure when ρ_t is close to 1, and there is strong selling pressure when it is close to -1 . A critique to using *OFI* to estimate market impact is that we have to use the mid-prices returns. Thus, we run regression (2) using log-price returns (of realized prices (We also run *OFI* regression using market price instead of mid-prices, but as expected the regression provides no significant results, see also Cont et al.²⁸)) and by substituting *OFI* with ρ .

Table 4 reports the time average estimates of market-impact coefficients among the trading periods. In contrast to the previous case, the market impact estimates also seem to deteriorate for self-impact terms. Furthermore, we observe now significant positive cross-impact effects of volume imbalance of asset 2 to returns of asset 1 in T2 treatment with market-neutral orders following asset 1. In general, we observe a persistent self-market impact effect for the value asset, while the self-impact of asset 1 results to be quite oscillatory.

We then explore the excess bids as another liquidity measure. Following Selten and Neugebauer,³⁸ we also test the predictive model of excess bids, i.e., we test the predictability power of the excess bids variable at period t on

asset return of the next period $t + 1$. The excess bids during the trading period t for agent j on asset i is defined as the difference in number of submitted bids $L_{t,i,j}^b$ minus offers $L_{t,i,j}^a$, which in our market design correspond to the number of limits buy and sell orders, respectively (see Smith et al.¹ and Selten and Neugebauer³⁸):

$$X_{t,i,j} = L_{t,i,j}^b - L_{t,i,j}^a$$

If we account also for market orders, we obtain, on the individual level, the excess bids after market clearing:

$$X'_{t,i,j} = (L_{t,i,j}^b - L_{t,i,j}^a) - (M_{t,i,j}^b - M_{t,i,j}^a)$$

where a market buy (sell) order for agent j means that j purchases (sales) the asset at the best ask (bid). Furthermore, if j places a market buy order and k is the seller, then we account the market clearing only for the excess bids agent j . Therefore, following Selten and Neugebauer,³⁸ we aggregate the excess bids measure at level of traders, i.e., we compute the average among agents $X'_{t,i} = 1/J \sum_{j=1}^J X'_{t,i,j}$. Thus, we run regression (2) at level of trading periods t among the simulations substituting *OFI* with excess bids X' . Moreover, we test the predictive hypothesis of Selten and Neugebauer³⁸ in our market setting, where using their notation, we set to zero the risk-free rate by regressing $r_{t+1,i,k}$ on $X'_{t,i,k}$. In order to avoid confusion, we call the first regression as “Market Impact Regression” and the second one “Predictive Regression.” Therefore, for each treatment, we compute unique market impact estimates. Table 9 shows the results. In contrast to previous cases, we obtain disparate results among T2 specifications (see, for example, R^2 measures).

Overall, these previous analyses show that the unique, consistent liquidity measure that provides more evident and significant market impact coefficients is the *OFI* measure.

Table 9. Market impact estimates using excess bids.

	Predictive regression				Market impact regression		
	Estimate	p-value	Adjusted R ²		Estimate	p-value	Adjusted R ²
				T2			
λ_{11}	0.388	0.008	0.131	λ_{11}	0.538	0.000	0.279
λ_{12}	0.367	0.005		λ_{12}	0.480	0.000	
λ_{21}	-0.123	0.087	0.147	λ_{21}	0.303	0.000	0.277
λ_{22}	-0.225	0.001		λ_{22}	0.496	0.000	
				T2-D1			
λ_{11}	0.008	0.960	0.215	λ_{11}	0.071	0.660	0.481
λ_{12}	0.116	0.265		λ_{12}	0.078	0.480	
λ_{21}	0.291	0.000	0.067	λ_{21}	-0.260	0.000	0.187
λ_{22}	-0.408	0.000		λ_{22}	0.590	0.000	
				T2-M1			
λ_{11}	0.580	0.000	0.164	λ_{11}	0.850	0.000	0.440
λ_{12}	0.267	0.013		λ_{12}	0.390	0.000	
λ_{21}	0.412	0.000	0.063	λ_{21}	0.124	0.204	0.034
λ_{22}	0.195	0.000		λ_{22}	0.207	0.000	

We estimate a predictive regression of r_{t+1} on X_t' and a contemporaneous regression of r_t on X_t' .

p-values are computed using robust HAC standard errors.

T2 refers to T2 treatments with independent orders, T2-D1 refers to T2 treatments with directional orders, and T2-M1 refers to T2 treatments with market-neutral orders.


Acknowledgement

The authors thank Oliver Kirchkamp and Stefan Palan for useful comments and suggestions.

Funding

The authors acknowledge support from the project “How good is your model? Empirical evaluation and validation of quantitative models in economics” funded by the Program for Research Projects of National Interest (PRIN) grant no. 20177FX2A7.

ORCID iD

Caterina Giannetti  <https://orcid.org/0000-0003-2962-3087>

References

- Smith VL, Suchanek GL and Williams AW. Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica* 1988; 56: 1119–1151.
- Caginalp G, Ilieva V, Porter D, et al. Do speculative stocks lower prices and increase volatility of value stocks. *J Psychol Financ Mark* 2002; 3: 118–132.
- Caginalp G and Ilieva V. The dynamics of trader motivations in asset bubbles. *J Econ Behav Organ* 2008; 66: 641–656.
- Kirchler M, Huber J and Stöckl T. Thar she bursts: reducing confusion reduces bubbles. *Am Econ Rev* 2012; 102: 865–883.
- Baghestanian S, Lugovskyy V and Puzzello D. Traders' heterogeneity and bubble-crash patterns in experimental asset markets. *J Econ Behav Organ* 2015; 117: 82–101.
- Farjam M and Kirchkamp O. Bubbles in hybrid markets: how expectations about algorithmic trading affect human trading. *J Econ Behav Organ* 2018; 146: 248–269.
- Nosek BA, Ebersole CR, DeHaven AC, et al. The preregistration revolution. *Proc Natl Acad Sci USA* 2018; 115: 2600–2606.
- Duffy J and Ünver MU. Asset price bubbles and crashes with near-zero-intelligence traders. *Econ Theory* 2006; 27: 537–563.
- CFTC-SEC. Findings regarding the market events of May 6, 2010. Report, 2010, <https://www.sec.gov/files/marketevents-report.pdf>
- Kirilenko A, Kyle AS, Samadi M, et al. The flash crash: high-frequency trading in an electronic market. *J Financ* 2017; 72: 967–998.
- da Gama Batista J, Massaro D, Bouchaud J-P, et al. Do investors trade too much? A laboratory experiment. *J Econ Behav Organ* 2017; 140: 18–34.
- Brogaard J, Carrion A, Moyaert T, et al. High frequency trading and extreme price movements. *J Financ Econ* 2018; 128: 253–265.
- Bouchaud J-P, Gefen Y, Potters M, et al. Fluctuations and response in financial markets: the subtle nature of “random” price changes. *Quant Financ* 2004; 4: 176–190.
- Gerig AN. *A theory for market impact: how order flow affects stock price*. PhD Thesis, University of Illinois at Urbana-Champaign, Champaign, IL, 2007.
- Bouchaud J-P, Farmer JD and Lillo F. How markets slowly digest changes in supply and demand. In: Hens T and Schenk-Hoppé KR (eds) *Handbook of financial markets: dynamics and evolution*. Amsterdam: Elsevier, 2009, pp. 57–160.
- Cespa G and Foucault T. Illiquidity contagion and liquidity crashes. *Rev Financ Stud* 2014; 27: 1615–1660.
- Alfonsi A, Klöck F and Schied A. Multivariate transient price impact and matrix-valued positive definite functions. *Math Oper Res* 2016; 41: 914–934.

18. Schneider M and Lillo F. Cross-impact and no-dynamic-arbitrage. *Quant Financ* 2019; 19: 137–154.
19. Cordoni F and Lillo F. Instabilities in multi-asset and multi-agent market impact games. *arXiv preprint arXiv:2004.03546*, 2020.
20. Chordia T, Roll R and Subrahmanyam A. Commonality in liquidity. *J Financ Econ* 2000; 56: 3–28.
21. Pasquariello P and Vega C. Strategic cross-trading in the us stock market. *Rev Financ* 2015; 19: 229–282.
22. Haruvy E and Noussair CN. The effect of short selling on bubbles and crashes in experimental spot asset markets. *J Financ* 2006; 61: 1119–1157.
23. Wan HA and Hunter A. On artificial adaptive agents models of stock markets. *Simulation* 1997; 68: 279–289.
24. Thompson JR and Wilson JR. Agent-based simulations of financial markets: zero-and positive-intelligence models. *Simulation* 2015; 91: 527–552.
25. Aydin NS. Reinforcement-learning-based optimal trading in a simulated futures market with heterogeneous agents. *Simulation* 2022; 98: 321–333.
26. Gode DK and Sunder S. Allocative efficiency of markets with zero-intelligence traders: market as a partial substitute for individual rationality. *J Polit Econ* 1993; 101: 119–137.
27. Gode DK and Sunder S. Human and artificially intelligent traders in computer double auctions. In: Carley KM and Prietula MJ (eds) *Computational organization theory*. Hillsdale, NJ: Lawrence Erlbaum Associates, 1994, pp. 241–262.
28. Cont R, Kukanov A and Stoikov S. The price impact of order book events. *J Financ Economet* 2014; 12: 47–88.
29. Fink J, Palan S and Theissen E. Earnings autocorrelation and the post-earnings-announcement drift: experimental evidence. *Technical report, CFR working paper*, 2020.
30. Guéant O. *The financial mathematics of market liquidity: from optimal execution to market making*, vol. 33. Boca Raton, FL: CRC Press, 2016.
31. Guéant O, Lehalle C-A and Fernandez-Tapia J. Dealing with the inventory risk: a solution to the market making problem. *Math Financ Econ* 2013; 7: 477–507.
32. Ho T and Stoll HR. Optimal dealer pricing under transactions and return uncertainty. *J Financ Econ* 1981; 9: 47–73.
33. Avellaneda M and Stoikov S. High-frequency trading in a limit order book. *Quant Financ* 2008; 8: 217–224.
34. Li F, Chow T-M, Pickard A, et al. Transaction costs of factor-investing strategies. *Financ Anal J* 2019; 75: 62–78.
35. Palan S. A review of bubbles and crashes in experimental asset markets. *J Econ Surv* 2013; 27: 570–588.
36. Kyle AS. Continuous auctions and insider trading. *Econometrica* 1985; 53: 1315–1335.
37. Mertens LP, Ciacci A, Lillo F, et al. Liquidity fluctuations and the latent dynamics of price impact. *Quant Financ* 2022; 22: 149–169.
38. Selten R and Neugebauer T. Experimental stock market dynamics: excess bids, directional learning, and adaptive style-investing in a call-auction with multiple multi-period lived assets. *J Econ Behav Organ* 2019; 157: 209–224.
39. Cordoni F. Multi-asset bubbles equilibrium price dynamics. *arXiv preprint arXiv:2206.01468*, 2022.
40. Gatheral J. No-dynamic-arbitrage and market impact. *Quant Financ* 2010; 10: 749–759.
41. Tomas M, Mastromatteo I and Benzaquen M. How to build a cross-impact model from first principles: theoretical requirements and empirical results. SSRN 3567815, 2020, <https://arxiv.org/pdf/2004.01624.pdf>
42. Cartea A, Donnelly R and Jaimungal S. Enhancing trading strategies with order book signals. *Appl Math Financ* 2018; 25: 1–35.

Author biographies

Francesco Cordoni is a Postdoctoral Research Assistant at Royal Holloway, University of London—Department of Economics, under the research project “Information Content and Dissemination in High-Frequency Trading” funded by the Leverhulme Trust. He is also Visiting Research Scholar at Bayes Business School (formerly Cass)—City, University of London. From November 2020 to September 2022, he worked as a Post-Doc Researcher at the University of Pisa—Department of Economics and Management on the PRIN project entitled “How good is your model? Empirical evaluation and validation of quantitative models in economics” jointly with Scuola Superiore Sant’Anna. He received the PhD in Financial Mathematics at Scuola Normale Superiore. His main research interests are quantitative finance, econometrics for financial markets, market microstructure, and causal inference.

Caterina Giannetti is Associate Professor at the Department of Economics and Management at University of Pisa (Italy), an affiliated researcher at Center E. Piaggio, University of Pisa, and affiliated researcher at Oliver Kirchcamp’s Chair in Empirical and Experimental Methods, University of Jena (Germany). She obtained her PhD (Doctor *Europaeus*) at the Institute for Advanced Studies IMT Lucca in 2008. She spent research periods across different European Universities such as Tilburg University (the Netherlands), Oxford University, Royal Holloway (UK), Catholic University of Louvain (Belgium), University of Jena (Germany), and University of Bologna (Italy). Her main research field is on Empirical Finance, with an interdisciplinary focus on human–robot interaction and intersection with Experimental Economics.

Fabrizio Lillo is Full Professor of Mathematical Methods for Economics and Finance at the University of Bologna (Italy). Since July 2021, he also holds the chair (part time) of Mathematical Finance on Market Microstructure, Networks and Systemic Risk at the Scuola Normale Superiore in Pisa (Italy). Formerly he has been Associate Professor of Mathematical Finance at the Scuola Normale Superiore, where he led the Quantitative Finance group. He has also been Postdoc, External Faculty, and Professor at the Santa Fe Institute (USA). He received the PhD in

Physics at the University of Palermo (Italy) where he was also Assistant Professor. His main research topics are market microstructure, data science, econometrics, forecasting models, high-frequency trading, machine learning, portfolio optimization, risk management, trading strategies, and transaction cost analysis.

Giulio Bottazzi is a Full Professor of Economic Policy in the Social Science Department of the Scuola Superiore Sant'Anna in Pisa. He graduated from Parma University and holds a PhD in Theoretical Physics from Milano University. He is the author of several papers published in international journals. His research interests range from

industrial dynamics to financial economics, encompassing experimental economics (coordination and bounded rationality), economic geography (industrial districts and localized spillovers), theoretical and empirical aspects of market interactions (firm growth and diversification; competition and technical efficiency), and applied econometrics (characterization of extreme events). He has been the Principal Investigator and Co-Investigator of projects financed by the Italian Ministry of University and Research, the European Commission, and the Tuscany Regional Council, both in partnership with other research institutions and private companies.